$\Delta x\Delta z$ and pointing into the phase of interest. The fractional rate of change of area of surface element $\Delta x\Delta z$

$$\frac{1}{\Delta x\Delta z}\left(\frac{\partial S}{\partial t}\right)_{\text{avg}} = [\Delta x(v_z|_{z+\Delta z} - v_z|_z) + \Delta z\,(v_x|_{x+\Delta x} - v_x|_x)]\,\frac{1}{\Delta x\Delta z} \qquad (A1)$$

By taking the limit as (Δx, Δz) approach zero, this expression may be rewritten for any point u, w in the interfacial surface as

$$\left(\frac{\partial \ln S}{\partial t}\right)_{u,w} = \frac{\partial v_x}{\partial x} + \frac{\partial v_z}{\partial z} \qquad (A2)$$

For an incompressible fluid this expression may be rewritten with the aid of the continuity relation to obtain

$$\left(\frac{\partial \ln S}{\partial t}\right)_{u,w} = -\frac{\partial v_y}{\partial y} \qquad (A3)$$

Since $(\partial \ln S/\partial t)_{u,w}$ is independent of y, we may write for negligible net transfer across the interface

$$v_y = -y(\partial \ln S/\partial t)_{u,w} \qquad (2)$$

For curved surfaces this relation will be only approximately correct and should only be used for small y, more specifically for y small compared with the local radii of curvature. This is consistent with the limitations on the penetration theory of mass transfer.

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The Evaporation of Drops of Pure Liquids at Elevated Temperatures: Rates of Evaporation and Wet-Bulb Temperatures

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The rates of evaporation and the wet-bulb temperatures have been correlated for drops of pure liquids evaporating in streams of high-temperature air. The four liquids studied were acetone, benzene, *n*-hexane, and water. The drops were of the order of a millimeter in diameter and were suspended in a free jet of dry, vapor-free air that ranged in temperature from 27° to 340°C. Reynolds numbers ranged from 24 to 325. Corrections to the Nusselt number to account for the heat lost to the outwardly diffusing vapor ranged up to about 35%.

The correlation of the phenomena characterizing the evaporation of liquid drops under conditions leading to high heat and mass fluxes would be of interest in many fields of commercial importance, but probably the greatest utility of such a correlation would be in relation to spray-drying and droplet-combustion studies. The present paper presents a simplified approach to such a correlation, being concerned solely with the evaporation of isolated pure liquid drops at atmospheric pressure.

DROP EVAPORATION RATES

A heat balance taken on a differential shell about a spherically symmetrical model of a drop undergoing steady state evaporation leads to the differential equation (constant film properties)

$$\frac{d^2T}{dr^2} + \left[\frac{2}{r} - \frac{w\,c_{P,v}}{4\pi\,k\,r^2}\right]\frac{dT}{dr} = 0 \qquad (1)$$

with the boundary conditions of $r = R_1$, $T = T_1$, and at $r = R_0$, $T = T_0$. This equation implies that k, the thermal conductivity of the film, is constant. In the general case, however, k will be a function of both composition and temperature. In order to maintain the simple form of Equation (1), it is therefore necessary to define an effective average thermal conductivity of the film, to be evaluated at some appropriate composition and temperature which are as yet undefined. By designating this average thermal conductivity of the film as k_f, the solution of Equation (1) is (*13*)

$$\frac{T - T_1}{T_0 - T_1} = \frac{\exp(-a'(R_1/r)) - \exp(-a')}{\exp(-a'(R_1/R_0)) - \exp(-a')} \qquad (2)$$

Equation (2) may now be differentiated to obtain the temperature gradient at the drop surface:

$$\left.\frac{dT}{dr}\right|_{R_1} = \frac{a'\,\Delta T}{R_1}\,[\exp\,(a'\,(1 - R_1/R_0)) - 1]^{-1} \qquad (3)$$

The expression for the heat balance at the drop surface is

$$k_f\,A_1\left.\frac{dT}{dr}\right|_{R_1} = w\,\Delta H_s \qquad (4)$$

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In the limiting case in which the heat required to raise the temperature of the outwardly diffusing vapor (from the wet bulb to the dry bulb) may be neglected relative to the total heat supplied, the equation $h_1{}^o A_1 \Delta T = k_f A_f (\Delta T/\Delta R)$ may be used to define a wind factor for heat transfer as

$$f^o{}_{HT} = \frac{1}{2} N^o{}_{Nu} = \frac{h_1{}^o D_1}{2 k_f} = [1 - R_1/R_0]^{-1} \qquad (5)$$

Combination of Equations (3), (4), and (5) gives the rate of drop evaporation as

$$\left[-\frac{dD^2}{dt} \right] = \left[\frac{8 k_f \Delta T f^o{}_{HT}}{\rho_l \Delta H_s} \right] \left[\frac{a}{e^a - 1} \right] \qquad (6a)$$

In the limit in which the perturbation of the heat transfer process due to the simultaneous transfer of mass may be neglected, the term $(a/(e^a - 1))$ goes to unity, and the rate becomes

$$\left[-\frac{dD^2}{dt} \right]^o = \left[\frac{8 k_f \Delta T f^o{}_{HT}}{\rho_l \Delta H_s} \right] \qquad (6b)$$

Now the actual evaporation rate is proportional to N_{Nu}, while the rate which would be anticipated in the absence of the mass transfer perturbation of the heat transfer process is proportional to $N^o{}_{Nu}$, so that from Equations (6)

$$\frac{(-dD^2/dt)}{(-dD^2/dt)^o} = \frac{N_{Nu}}{N^o{}_{Nu}} = \frac{a}{e^a - 1} \qquad (7)$$

If a' in Equation (3) is replaced by its equivalent value from Equation (2), Equation (6*a*) may be rewritten as

$$\left[-\frac{dD^2}{dt} \right] = \left[\frac{8 k_f \Delta T f^o{}_{HT}}{\rho_l \Delta H_s} \right] \left(\frac{1}{B} \right) \ln(1 + B) \qquad (8)$$

Equation (7) may be rewritten in the more useful form

$$\frac{N_{Nu}}{N^o{}_{Nu}} = \left(\frac{1}{B} \right) \ln (1 + B) \qquad (9)$$

DROP TEMPERATURES

Combination of the mass transfer analog of Equation (5) with Equation (17.2-21) of reference *4* permits a wind factor for mass transfer to be defined as

$$f_{MT} = \frac{N_{Sh}}{2} = -\frac{\mathcal{R}\, T_f \,\rho_l\, (-dD^2/dt)}{8\, P\, M_v\, \mathcal{D}_v \ln \left[\dfrac{P - p_{v,1}}{P - p_{v,0}} \right]} \qquad (10)$$

Combination of Equations (3), (4), and (10) then gives an equation for the wet-bulb depression:

$$(T_0 - T_1) = \frac{\Delta H_s}{c_{p,v}} \left\{ \left[\frac{P - p_{v,1}}{P - p_{v,0}} \right]^{-b} - 1 \right\} \qquad (11)$$

where

$$b = \frac{f_{MT}\, M_v\, P\, \mathcal{D}_v\, c_{P,v}}{f^o{}_{HT}\, k_f\, \mathcal{R}\, T_f}, \text{ dimensionless} \qquad (12)$$

Here, as in the preceding section, the physical and transport properties in the film must be evaluated at the average film temperature and composition.

THE AVERAGE FILM TEMPERATURE AND COMPOSITION

The volume-average film temperature, defined as

$$T_f = \left[\frac{3}{R_0{}^3 - R_1{}^3} \right] \int_{R_1}^{R_0} T(r)\, r^2\, dr \qquad (13)$$

was used, with the average film concentration $(p_{v,f} - p_{v,1})/(p_{v,0} - p_{v,1})$ being defined analogously. Over the range of conditions encountered in this study, however, the dimensionless film temperature variable m, defined as

$$m = \frac{T_f - T_1}{T_0 - T_1} \qquad (14)$$

was calculated as 0.60 ± 0.02, root-mean-square. The use of $m = 0.60$ for the calculation of both the average film temperature and average film composition is therefore recommended.

LITERATURE REVIEW

The correlation of Ranz and Marshall (*34*)

$$\left. \begin{aligned} N^o{}_{Nu} &= 2.0 + 0.60\, N_{Pr}{}^{1/3}\, N_{Re}{}^{1/2} \\ N_{Sh} &= 2.0 + 0.60\, N_{Sc}{}^{1/3}\, N_{Re}{}^{1/2} \end{aligned} \right\} \qquad (15)$$

for heat and mass transfer at low fluxes was originally based on data taken within the range $2 < N_{Re} < 200$. A subsequent critical review (*8*) of the available literature data indicated that this correlation was valid for Reynolds numbers up to at least about 1,000. This correlation will therefore be used as a starting point for the interpretation of the data obtained in the present study.

Several analytical developments (*2, 22, 24, 33, 38, 40*) are more or less similar to that presented above, and they will not be discussed here.

Tanasawa and Kobayashi (*42*) have analyzed the problem of unsteady state drop evaporation, and their results were later compared with the experimental data of Kobayashi (*20, 21*). Working with a series of pure liquids and petroleum fractions, this latter author photographically recorded the evaporation of drops suspended on a quartz fiber and introduced into a furnace. Nishiwaki (*25*) made a similar study and also made measurements in forced convection (*26*). El Wakil, Uyehara, Myers, and co-workers (*5, 9, 10, 30*) have made an intensive study of the phenomena accompanying the evaporation of liquid drops suspended in an air stream.

Maisel and Sherwood (*23*) measured the rates of evaporation of liquids from porous spheres to fluids flowing in a duct at moderate temperatures. Ingebo (*18, 19*) measured the vaporization rates of several pure liquids from a wetted-cork sphere held in a hot air stream ($0.38 < (1/B) \ln (1 + B) < 1.0$, $1{,}100 < N_{Re} < 2{,}270$). His experimental techniques were unrefined, however, and the accuracy of his results has been questioned (*3, 8*). Petrazhitskiy (*29*) measured the rate of evaporation of water from a porous sphere into a flue gas at 2,800°C. by using a method similar to that of Ingebo. He was, however, primarily concerned with the measurement of wet-bulb temperatures and droplet evaporation rates at high pressure as well as at high temperatures.

Topps (*43*), Saad (*37*), and Apashev and Malov (*1*) measured the rates of vaporization of drops falling freely through a furnace. Simpson (*17, 39*) studied the evaporation characteristics of liquid fuel drops projected upward into a furnace. Hall and Diederichsen (*15*) conducted studies along similar lines. Wood, Rosser, and Wise (*44*) were primarily concerned with droplet combustion.

Gohrbandt (*14*) studied the sublimation and combustion of camphor and anthracene spheres mounted in a hot air stream. Radusch (*31*), Richardson (*35*), Fedoseev and Polishchuk (*11, 12*), and Ranz (*32*) measured the rates of evaporation of drops suspended in streams of hot air. In addition, Richardson recorded vaporization rates for drops in free fall, and Fedoseev and Polishchuk made free convection measurements.

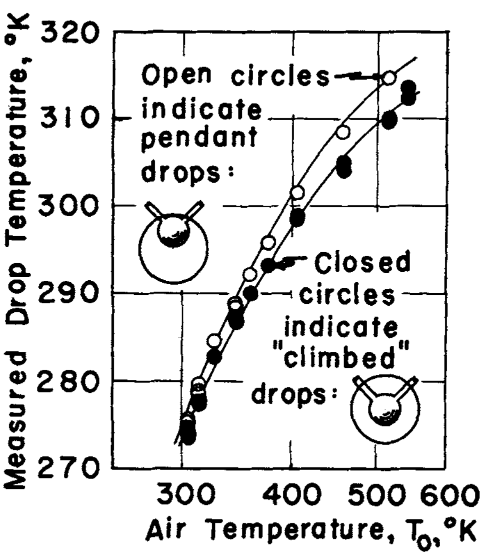

Fig. 1. Experimental wet-bulb temperatures for a benzene drop; raw data for a nominal velocity of 100 cm./sec.

Schluender (*38*) measured the rates of evaporation of water drops suspended in a chamber of moist air ($0.61 < (1/B) \ln (1 + B) < 0.94, 0.3 < N_{Gr} < 25$). He found that the rate of convective heat transfer was not affected by the magnitude of the Grashof number.

Hoffman and Gauvin (*16*) measured the rates of evaporation from droplets suspended in a quiescent atmosphere of superheated vapor ($0.69 < (1/B) \ln (1 + B) < 1.0, 1.2 < N_{Gr} < 108$). Pei and Gauvin (*27*) extended these measurements by using large porous spheres, the surfaces of which were wetted with tempered liquid [$0.23 < (1/B) \ln (1 + B) < 0.93, 210 < N_{Gr} < 2.2(10^5)$]. In neither investigation was the Grashof number found to influence the rate of convective heat transfer.

The 9-in. diameter spherical chamber used by Hoffman and Gauvin and by Pei and Gauvin was modified by Pei, Narasimhan, and Gauvin (*28*) by the installation of internal ductwork to permit a continuous stream of superheated vapor to be passed over the wetted spheres. These latter authors were thus able to explore the regions of forced, free, and forced and free convection.

APPARATUS AND PROCEDURE

The air supply system used and described by Ranz (*32, 34*) and by Charlesworth (*6*) was rebuilt for use in the present investigation. The drops to be studied were mounted, one at a time, near the axis of a 1-in. diameter free jet of dry, vapor-free air projected upward. A swinging baffle was used to deflect the air jet between runs. The measured intensity of turbulence in the jet was less than one-third of 1%.

Reynolds numbers ranged from 24 to 325. Jet temperatures, ranging from 27° to 340°C., were recorded with calibrated 3-mil constantan-manganin thermocouples. Corrections for radiant heat transfer to the tied or butt-welded sensing elements were estimated analytically.

Rates of evaporation data were obtained by suspending the drops from glass filaments and recording the time-rate-of-change of drop size with a 16-mm. Kodak Cine II Special Camera driven by a synchronous motor. Drop sizes were recorded by projecting, frame-by-frame, the developed motion picture film and by measuring the maximum horizontal dimensions of the projected images on a sheet of paper which had been previously calibrated with the aid of photographs taken of a stage micrometer. The scale was about 2-7/16 in./mm., and the precision of the measurements was equivalent to ± 0.01 mm.

Substitution of Equations (15) into Equations (5) and (6) shows that at a constant T_0 $D^{3/2}$ should be approximately linear with t at high Reynolds numbers and that D^2 should be approximately linear with t at low Reynolds numbers. Therefore drop diameters were calculated to $D^{3/2}$ and to D^2, and plots were made of both $D^{3/2}$ against t and D^2 against t. Slopes of the curves were taken for the most part at 0.02-cm. increments of D and converted to a common basis by the relationship $(-dD^2/dt) = 4/3\sqrt{D}\ (-dD^{3/2}/dt)$. If for a given point the value of $(-dD^2/dt)$ obtained from a plot of $D^{3/2}$ against t differed from the value obtained from a plot of D^2 against t by more than 3% for water or more than 5% for acetone, benzene, and hexane, the data point was discarded. The pairs of values which met these criteria of acceptance were averaged and recorded. A total of four hundred and ninety-five data points was so obtained.

Steady state drop temperatures were recorded by suspending the drops from calibrated beaded element constantan-manganin thermocouples. Replicate measurements were made with geometrically similar thermocouples fabricated from 5-, 3-, and 1-mil wires. An extrapolation of these data then permitted an estimate to be made of the error introduced into the measurements by the conduction of heat through the thermocouple wires to the sensing elements.

The jet temperatures were recorded directly with a potentiometer. The thermal emf's from the thermocouples indicating the wet-bulb temperatures were amplified and sent through bucking and attenuation units to a high-speed strip-chart recorder.

SURVEY OF THE EFFECTS OF VARIOUS PARAMETERS

Drop Shape

A drop suspended from a filament bead will, in general, tend to assume a pear shape. In the present study, the effect of this type of distortion was minimized by directing the air stream onto the drop from below, so that the distortion was primarily confined to the lee side, and

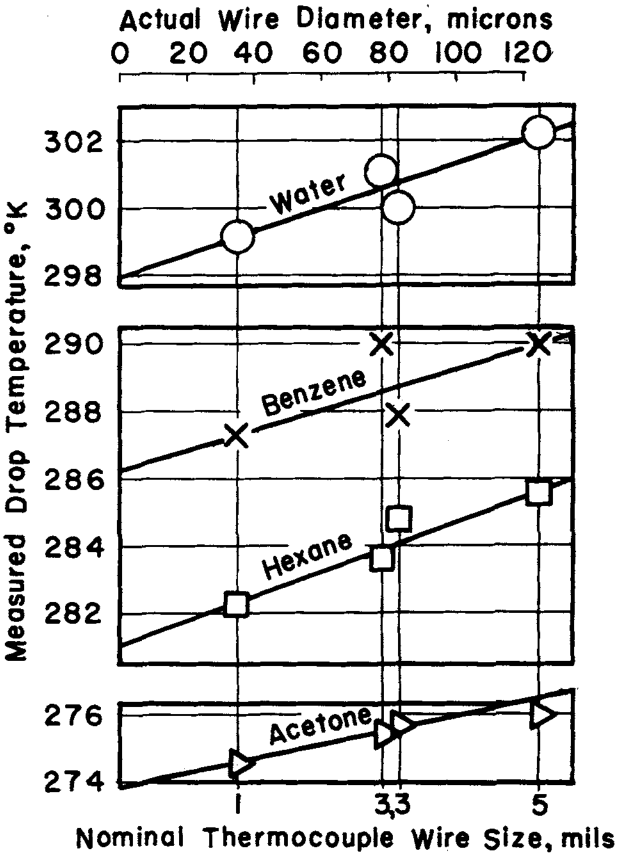

Fig. 2. Extrapolation of the wet-bulb temperature data for a 350°K., 100 cm./sec. air stream (raw data). All data are for climbed drops.

by restricting measurements to the range of drop sizes over which the drop was essentially spherical.

Drop Distortion

At high air velocities, the drops tended to rotate in a horizonal plane about the filament bead, making rate of evaporation measurements impossible. This mode of distortion suggests that a large drop, falling freely, would tend to follow a helical rather than a straight-line path. At elevated temperatures the water drops were observed at intervals to oscillate very rapidly between prolate and oblate spheroidal forms. It is believed that this latter mode of drop distortion was the result of a resonance phenomenon; it was observed only when the natural frequency of vibration of the drop (or a harmonic) corresponded to the frequency of eddy detachment from the drop wake. Within the precision of the measurements made in this study, however, this latter mode of vibration had no effect upon the rate of drop evaporation.

Internal Drop Circulation

The internal circulation in a liquid drop effectively reduces the thickness of the boundary layers and increases the values of the overall Nusselt and Sherwood numbers above those which would be expected in the absence of circulation. All of the experimental data collected in this study were corrected for this effect by use of the semi-theoretical correlation of Conkie and Savic (7). The indicated corrections ranged up to 8%.

The Presence of Nonvolatile Impurities

The presence of nonvolatile impurities will result in the recorded rates of evaporation being too low. If the impurity is surface active, even a minute quantity may lead to erroneous results.

Chemically pure acetone and benzene were used in this study. The hexane was research grade, and the water was carefully repurified. Errors due to the presence of nonvolatile impurities were probably negligible.

Free Convection

Errors due to free convection were negligible.

Radiation

The radiant energy absorbed by the drops was estimated by superimposing the infrared absorption curves for the liquids on the flux-density curves for a black-body emitter, both plotted against wavelength. Analysis on this basis showed that a 100-μ thick layer of any one of the four liquids would absorb essentially all of the incident radiation. Since the drops studied here were all of the order of 1,000 μ in diameter, the absorptivity was taken to be constant and equal to 0.95. Schluender (38) used the same method but obtained smaller values because his calculations were based on a layer of liquid 10 μ thick.

The rate of evaporation data were corrected for radiant heat transfer; these corrections were small. Corrections to the wet-bulb temperatures for radiant heat transfer were negligible.

Variation in the Position of the Drop Suspending Filament

The velocity profile of the jet was nominally flat (32), so that small variations in the horizontal location of the drop had no effect on the measurements. And, since the jet persisted for several centimeters beyond the nozzle, small variations in the vertical position of the drop were similarly found to have little effect on the measurements.

Drop Size

If suitable corrections for internal drop circulation and drop distortion are applied to the measurements taken on drops of the order of a millimeter in diameter, the results should be directly applicable to drops of the order of a few hundred microns in diameter simply by the use of the Reynolds number criterion of dynamic similarity. The results of the present study, having been so corrected, should thus be directly applicable to the evaporation of drops of the size range of interest in spray drying and related fields.

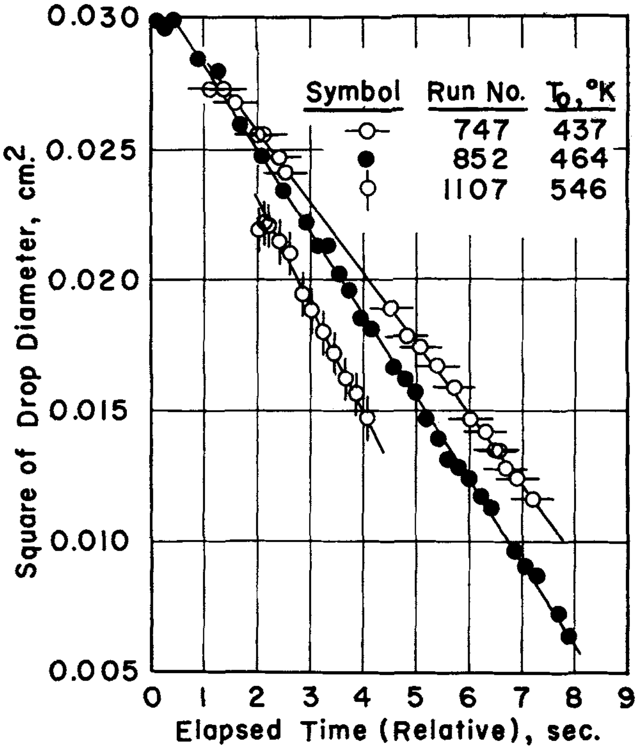

Fig. 3. Typical evaporation data. Hexane drops in a nominal 100 cm./sec. air stream.

EXPERIMENTAL RESULTS

A drop attached to the 1-mil beaded-element thermocouple was pendant, but as evaporation proceeded a drop size was usually reached at which the capillary and drag forces over balanced the gravitational force with the result that the drop climbed the wires. This reorientation of the drop relative to the temperature sensing element resulted in an abrupt change in the depth of immersion of the thermocouple in the liquid, so that two separate steady state wet-bulb temperatures were recorded for the drop. Thus, except for the water drops, two distinct wet-bulb temperature curves were obtained with the 1-mil thermocouple, as illustrated in Figure 1. Water apparently did not exhibit the same tendency to wet the thermocouple wires as did the organic liquids, so that no climbed measurements were obtained on the water drops.

When the drop temperatures were recorded with the 3- and 5-mil thermocouples, the larger wires gave rise to greater capillary forces; these, in turn, usually caused the drops to climb the wires before the steady state temperatures could be recorded for the pendant drops. The measurements taken with these larger thermocouples therefore refer in general to drops that were in some variation of the climbed configuration, and the effective depth of immersion was some function of the size that the drop had at the time that the temperature was recorded. Due to the greater amount of heat conducted by these larger thermocouple wires, the recorded drop temperatures were significantly influenced by this depth of immersion, and the precision of the measurements taken with these larger thermocouples was correspondingly less than the precision of the measurements taken with the 1-mil thermocouple.

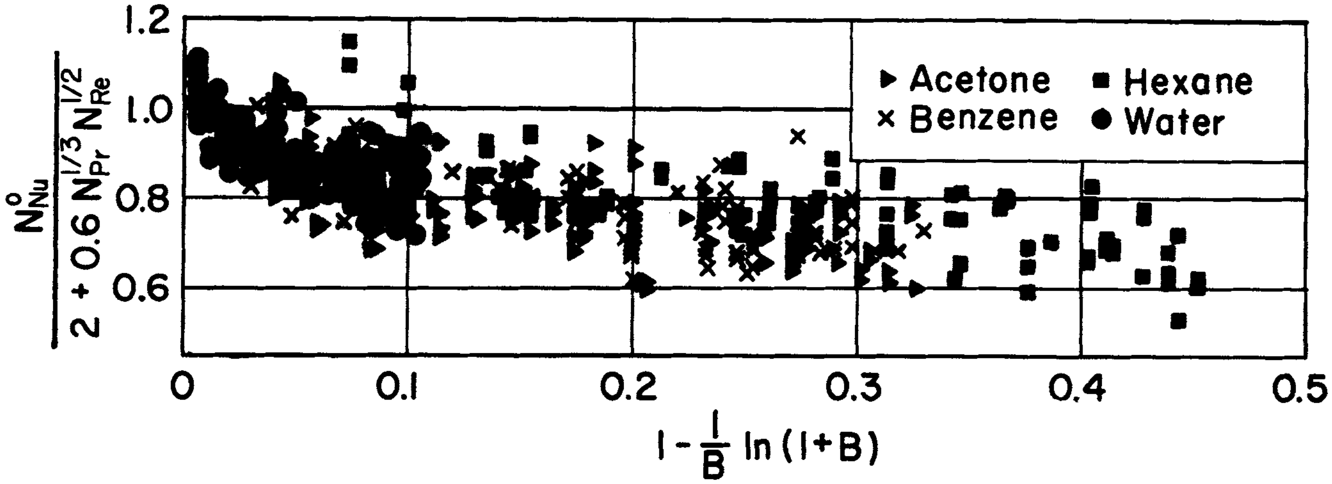

Fig. 4. Raw experimental rate of evaporation data.

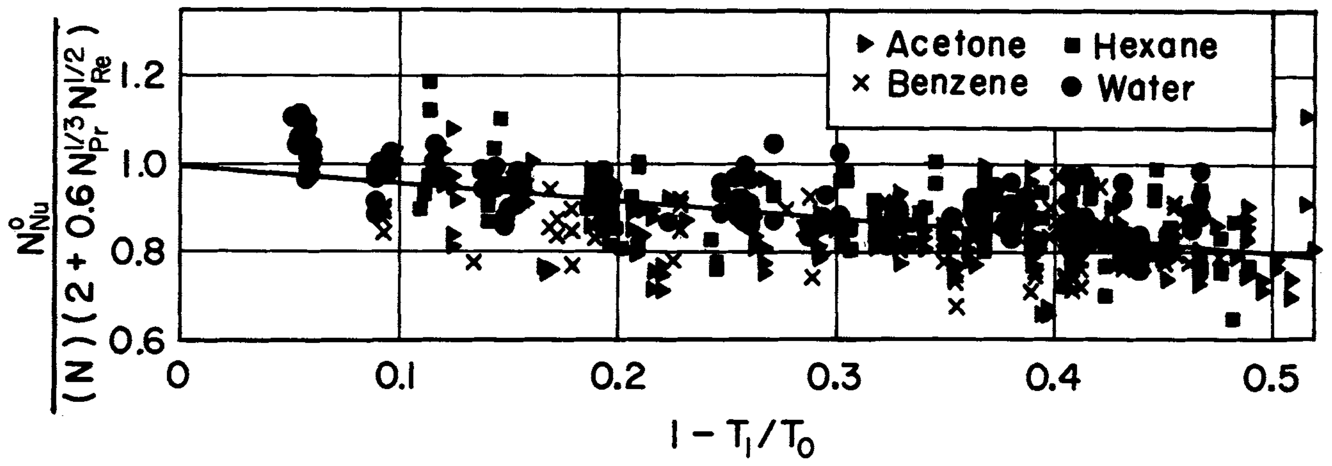

Fig. 5. Correlation of the rate of evaporation data according to Equation (16). Dependence upon the factor $(1 - T_1/T_0)$.

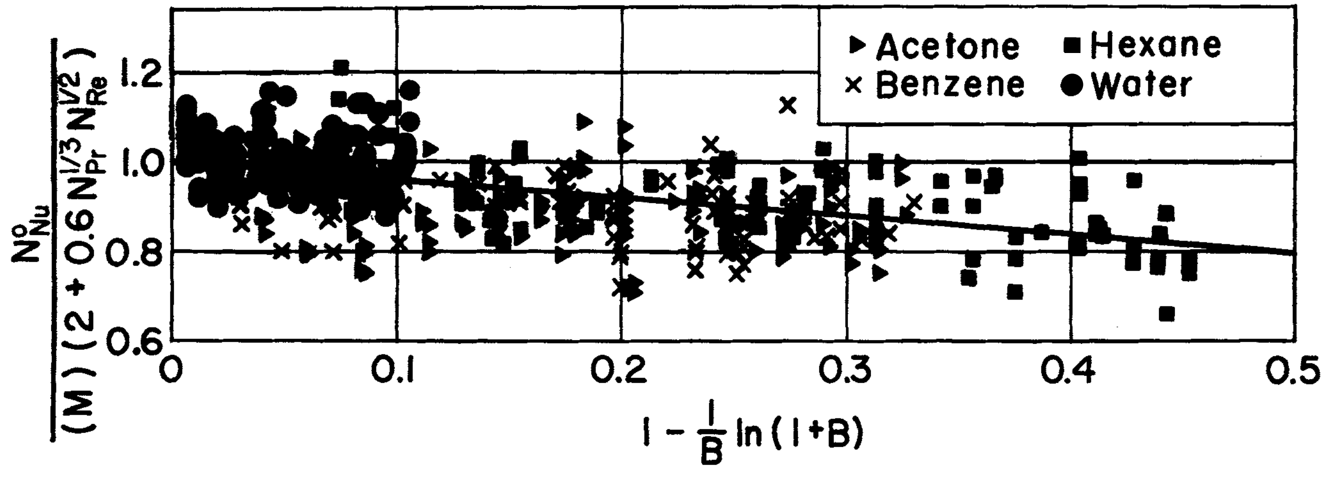

Fig. 6. Correlation of the rate of evaporation data according to Equation (16). Dependence upon the factor $(1/B) \ln (1 + B)$.

The extrapolation of typical wet-bulb temperature data to zero wire diameter is shown in Figure 2. Since the limited depth of immersion of the sensing element indicates that even the data taken with the 1-mil thermocouple on climbed drops may be systematically high, and since extrapolation against the first power of the wire diameter leads to the lowest plausible interpretation of the data, it is probable that the true wet-bulb temperatures lie between these two limits. The uncertainty here is small, however, and is within the precision with which the recursion formulas could be fitted to the data.

Typical rate of evaporation data are presented in Figure 3.

DISCUSSION OF RESULTS

The mass transfer data proved to be exceedingly sensitive to small uncertainties in the values estimated for the wet-bulb temperatures. For this reason, the evaporation rate measurements were correlated in terms of the heat transfer rates rather than in terms of the mass transfer rates. These heat transfer data, as plotted in Figure 4, are seen to fall as much as perhaps 35% below the values which were predicted from the film theory analysis presented above. This discrepancy between theory and observation can be most simply reconciled by the introduction of two empirical correction factors, the first of which involves the group $(1/B)\ \ln\ (1 + B)$ and thus accounts for the heat taken up by the outwardly diffusing vapor. The second factor involves the group T_1/T_0 and thus accounts, at least to a first approximation, for the fact that the film properties were considered to be constant in the theoretical development while they are functions of temperature in the actual system. The expression for the Sherwood number was then obtained by analogy, omitting of course the correction factor involving the $(1/B)$ $\ln\ (1 + B)$ group. The correlation obtained by use of this approach is

$$N_{Nu} = M\, N\, (1/B)\ \ln\ (1 + B)\ (2.0 + 0.60\, N_{Pr}{}^{1/3}\, N_{Re}{}^{1/2}) \qquad (16)$$

and

$$N_{Sh} = M\, (2.0 + 0.60\, N_{Sc}{}^{1/3}\, N_{Re}{}^{1/2})$$

where

$$M = 1 - 0.40\ (1 - T_1/T_0),\ \text{and}$$
$$N = 1 - 0.40\ (1 - (1/B)\ \ln\ (1 + B))$$

and the term $(1 - T_1/T_0)$ is the psychrometric temperature ratio or Gukhman number (*41*). The factors M and N are strictly empirical and evolved from simple cross plots of the data. Figures 5 and 6 present the heat transfer data as functions of these two factors; the average deviation of the data from the values predicted by Equation (16) is ± 6%.

Equation (16) should be used only in the context of the values of the physical and transport properties and the definitions of the average film temperature and composition used in its development (*8*).

The correlation presented in Equation (16) was calculated by using recursion formulas for the wet-bulb temperatures based on the experimental data; the values of the wet-bulb temperatures calculated from Equation (11) were found to be in error by as much as 3°C. It was found, however, that the substitution of the product bE for b alone in Equation (11), where E is the empirical correction factor given in Table 1, permitted the calculation of best value wet-bulb temperature curves; that is, curves lying between those measured for the climbed drops on the 1-mil thermocouple and those obtained by extrapolation (Figure 2). Thus modified, Equation (11) was used to calculate psychrometric charts for the four liquids studied here. These are presented elsewhere (*8*).

TABLE 1. VALUES OF THE EMPIRICAL FACTOR E
$E = m' + n\ (1 - T_1/T_0)$

	m'	n
Acetone	1.16	−0.08
Benzene	1.03	0
Hexane	1.23	−0.56
Water	0.83	0.34

The chart obtained for water serves to provide experimental confirmation for a chart previously available (*45*).

CONCLUSIONS

1. At high rates of heat and mass transfer, the correlation of Ranz and Marshall (*34*) must be modified. A film theory analysis gave a first-order approximation to this correction, and subsequent empirical modification permitted the experimental data to be correlated to an average deviation of ± 6%.
2. The limited depths to which the thermocouple sensing elements could be immersed in the drops used in this study caused the experimental wet-bulb temperatures to be systematically high, but replicate measurements made with a range of thermocouple wire diameters permitted reasonable estimates of the true drop temperatures to be made by use of an extrapolation procedure.
3. Empirical modifications of the correlating equations permitted the calculation of high-temperature humidity charts for the four liquids studied here: acetone, benzene, hexane, and water.
4. Within the precision of the measurements, drop vibration was found to have no effect upon the rate of drop evaporation.

ACKNOWLEDGMENT

The thesis, the results of which are presented here, was sponsored by the General Foods Corporation and carried out under the supervision of Professor W. R. Marshall. Correlations were developed with equipment in the Numerical Analysis Laboratory at the University of Wisconsin. Recent work was correlated under a Faculty Research Grant at Oregon State University.

NOTATION

a = $(w\, c_{P,v}/4\, \pi\, k_f\, R_1\, f^o{}_{HT})$, dimensionless
a' = a $f^o{}_{HT}$
A_1 = area of the drop surface, sq.cm.
A_f = geometric mean film area $(A_1/A_0)^{1/2}$, sq.cm.
A_0 = area corresponding to the radius R_0, sq.cm.
b = dimensionless group defined by Equation (12)
B = $(c_{P,v}\, \Delta T/\Delta H_s)$, dimensionless
c_P = specific heat, cal./(g.)(°K.)
d = differential operator
$\mathfrak{D}_v$ = coefficient of diffusion of the vapor in the ambient gas, evaluated at the effective average film temperature, sq.cm./sec.
D, D_1 = diameter of the drop, cm.
E = empirical correction factor, dimensionless
$f^o{}_{HT}$ = wind factor for heat transfer, $N^o{}_{Nu}/2$, dimensionless
f_{MT} = wind factor for mass transfer, $N_{Sh}/2$, dimensionless
g = gravitational constant
h_1 = coefficient of heat transfer, cal./(sec.)(°K.)(sq. cm. of drop surface)
ΔH_s = latent heat of vaporization of the liquid, corrected for radiation and evaluated at T_1, cal./g. $\Delta H_s = \lambda - (Q_R/w)$

k = thermal conductivity, cal./(sq.cm.) (°K./cm.) (sec.)
m = $(T_f - T_1/T_0 - T_1)$
M = $1 - 0.40\ (1 - T_1/T_0)$
M_f = average molecular weight of the film, calculated at the effective average film composition
M_v = molecular weight of the vapor
N = $1 - 0.40\ [1 - (1/B) \ln (1 + B)]$
N_{Nu} = experimentally observed Nusselt number, $h_1 D_1/k_f$
N^o_{Nu} = Nusselt number for heat transfer which would be predicted in the absence of mass transfer perturbations
N_{Pr} = Prandtl number, $c_{P,f}\mu_f/k_f$
N_{Re} = Reynolds number, $D_1 v \rho_f/\mu_f$
N_{Sc} = Schmidt number, $\mu_f/\rho_f \mathcal{D}_v$
N_{Sh} = Sherwood number, defined by Equation (10)
p_v = partial pressure of the vapor, atm.
P = total ambient pressure, atm.
Q_R = heat transferred to the drop by radiation, cal./sec.
r = variable distance from the center of the drop, cm.
$\mathcal{R}$ = gas constant
R_1 = radius of the drop, cm.
R_f = geometric mean radius, $(R_1 R_0)^{1/2}$, cm.
R_0 = radial distance from the drop center to the point at which the film temperature becomes equal to the bulk temperature in the spherically symmetrical model, cm.
ΔR = $(R_0 - R_1)$
t = time, sec.
T = temperature, °K.
ΔT = $(T_0 - T_1)$
v = relative drop-fluid velocity, cm./sec.
V = volume of the drop, cc.
w = rate of drop evaporation, equal to $(\pi/4)\ \rho_l D_1 (-dD^2/dt)$, g./sec.

Greek Letters

λ = latent heat of vaporization of the liquid in the absence of radiation at T_1, cal./g.
μ_f = viscosity of the film, evaluated at the effective average temperature and composition, g./(cm.) (sec.)
ρ = density, g./cc.

Superscript

o = indicates that the mass transfer perturbation of the heat transfer rate is negligible

Subscripts

1 = interface or to interfacial conditions
f = film or to the effective average film conditions
l = liquid
0 = bulk phase or to the conditions existing in the bulk phase
v = vapor

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